

**AD-A220 284**

4

GL-TR-90-0021

ARL-TR-88-32

**Effect of GPS System Biases on Differential  
Group Delay Measurements**

David S. Coco  
Scott R. Dahlke  
Clayton Coker

University of Texas at Austin  
Applied Research Laboratories  
P.O. Box 8029  
Austin, TX 78713-8029

6 July 1988

Scientific Report No. 1

**SDTIC**  
**ELECTE**  
**APR 10 1990**  
**B**

**APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED**

**GEOPHYSICS LABORATORY  
AIR FORCE SYSTEMS COMMAND  
UNITED STATES AIR FORCE  
HANSCOM AIR FORCE BASE, MASSACHUSETTS 01731-5000**

"This technical report has been reviewed and is approved for publication"

  
GREGORY J. BISHOP  
Contract Manager

  
WILLIAM K. VICKERY  
Branch Chief

FOR THE COMMANDER

  
ROBERT A. SKRIVANEK  
Division Director

This report has been reviewed by the ESD Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS).

Qualified requestors may obtain additional copies from Defense Technical Information Center. All others should apply to the National Technical Information Service.

If your address has changed, or if you wish to be removed from the mailing list, or if the addressee is no longer employed by your organization, please notify GL/IMA Hanscom AFB, MA 01731. This will assist us in maintaining a current mailing list.

Do not return copies of this report unless contractual obligations or notice on a specific document requires that it be returned.

# UNCLASSIFIED

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE <b>6 July 1988</b>	3. REPORT TYPE AND DATES COVERED <b>Scientific Report No. 1</b>		
4. TITLE AND SUBTITLE  <b>Effect of GPS System Biases on Differential Group Delay Measurements</b>			5. FUNDING NUMBERS  <b>PE: 63424F PR 4643 TA 09 WU03</b>	
6. AUTHORS  <b>Coco, David S. Dahlke, Scott R. Coker, Clayton</b>			Contract: <b>MIPR-FY7121-87-03694</b>	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Applied Research Laboratories The University of Texas at Austin P.O. Box 8029 Austin, Texas 78713-8029</b>			8. PERFORMING ORGANIZATION REPORT NUMBER  <b>ARL-TR-88-32</b>	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) <b>Sponsored by: Geophysics Laboratory Hanscom AFB Massachusetts 01731-5000 Contract Manager: Gregory Bishop/LIS</b>		Monitored by: <b>Space &amp; Naval War- fare Systems Cmd Dept of the Navy Washington, DC 20363</b>		10. SPONSORING / MONITORING AGENCY REPORT NUMBER  <b>GL-TR-90-0021</b>
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION / AVAILABILITY STATEMENT  <b>Approved for public release; distribution is unlimited.</b>			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  <b>GPS satellite and receiver differential L1/L2 biases induce errors into the GPS derived ionospheric measurements of the total electron content. If these measurements are to be used for ionospheric calibration of other systems or for deriving ionospheric models, then these biases must be measured. These biases are a major limiting factor in the use of GPS ionospheric measurements for these and other similar applications.</b>  <b>A series of combined satellite plus receiver differential L1/L2 bias measurements has been made using phase smoothed pseudorange data from TI 4100 receivers at three sites operating over a five week period. These biases were estimated using a least squares estimation technique with a local second order polynomial ionospheric model. The analysis of these measurements shows that these biases can be estimated with an accuracy of 0.5 ns of differential delay, with improvements in the ionospheric model giving potential accuracies as low as 0.3 ns. The results indicate that these biases remain constant over a five week period. These measured biases will be compared with pre-launch calibration values and biases measured by other receivers.</b>				
14. SUBJECT TERMS <b>Global Positioning System    total electron content    group delay ionosphere</b>			15. NUMBER OF PAGES  <b>68</b>	
17. SECURITY CLASSIFICATION OF REPORT <b>UNCLASSIFIED</b>			18. PRICE CODE	
18. SECURITY CLASSIFICATION OF THIS PAGE <b>UNCLASSIFIED</b>		19. SECURITY CLASSIFICATION OF ABSTRACT <b>UNCLASSIFIED</b>		20. LIMITATION OF ABSTRACT  <b>SAR</b>

# UNCLASSIFIED

## TABLE OF CONTENTS

	<u>Page</u>
LIST OF FIGURES.....	v
LIST OF TABLES.....	vii
1. INTRODUCTION.....	1
2. IONOSPHERIC TOTAL ELECTRON CONTENT MEASUREMENTS USING GPS.....	5
3. LEAST SQUARES ESTIMATION OF L1/L2 BIASES.....	9
4. DESCRIPTION OF THE DATABASE - CETKE I CAMPAIGN.....	17
5. SPR L1/L2 BIASES ESTIMATED FROM THE CETKE I CAMPAIGN.....	21
6. COMPARISON OF ESTIMATED SPR L1/L2 BIASES WITH OTHER SOURCES.....	29
7. DISCUSSION OF THE IONOSPHERIC MODEL USED IN THE ESTIMATION TECHNIQUE.....	33
8. INTER-RECEIVER L1/L2 BIAS CALIBRATION.....	37
9. SUGGESTIONS FOR IMPROVEMENTS.....	41
10. SUMMARY.....	43
APPENDIX A - PHASE AVERAGING OF PSEUDORANGE MEASUREMENTS.....	45
APPENDIX B - USING SLANT TEC MEASUREMENTS TO DEVELOP THE IONOSPHERIC MODEL.....	51
REFERENCES.....	59



<b>Accession For</b>	
NTIS GRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
<b>Availability Codes</b>	
Dist	Avail and/or Special
A-1	

## LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
3.1	Obliquity Factor Model.....	12
3.2	Illustration of Ionospheric Intercept Point Geometry.....	13
3.3	Ionospheric Intercept Tracks.....	16
4.1	Clock Evaluation and Timekeeping Experiment (CETKE I) Ground Stations.....	18
4.2	Ionospheric Data Processing Flow.....	20
5.1	SPR L1/L2 Biases - Austin, Texas.....	22
5.2	SPR L1/L2 Biases Relative to Daily Means - Austin, Texas ...	23
5.3	SPR L1/L2 Bias Estimation Statistics for a 5 Week Period.....	24
5.4	SPR L1/L2 Biases - Adjusted to Austin's Receiver L1/L2 Bias.....	25
7.1	Vertical TEC from Ionosphere Model in ns dd.....	34
A.1	Combining Pseudorange and Phase Measurements .....	48
B.1	Bent Ionospheric Model of Electron Columnar Content.....	54
B.2	Obliquity Factor Model Geometry .....	56

## LIST OF TABLES

<u>Table</u>		<u>Page</u>
5.1	Estimated SPR L1/L2 Biases - Averages over a Five Week Period from CETKE I Data .....	26
5.2	Estimate of Receiver L1/L2 Bias Differences - CETKE I.....	28
6.1	L1/L2 SV Biases from Rockwell International Pre-launch Factory Tests.....	30
6.2	Comparison of SPR L1/L2 Biases.....	30
6.3	Comparison of SPR L1/L2 Biases Relative to PRN 9 .....	31
8.1	Receiver L1/L2 Bias Differences - Zero Baseline Tests .....	39

## I. INTRODUCTION

The measurement of ionospheric total electron content (TEC) from the Global Positioning System (GPS) is derived from the difference in the group delays of the L1 (1575.42 MHz) and L2 (1227.60 MHz) signals. The differential L1/L2 group delays, however, contain not only the differential delays induced by the ionosphere, but also the differential delays from the satellite vehicle (SV) and receiver hardware. If an accurate estimate of the absolute ionosphere TEC is required, then the SV and receiver differential group delays must be measured and removed from the measurement. In this report, the SV and receiver differential group delays are referred to as L1/L2 biases.

The GPS SVs and receivers introduce common timing biases for both L1 and L2 signals due to a variety of sources. These biases are usually not a concern to the general GPS user because they are absorbed into the receiver clock bias estimate. The difference between the L1 and L2 biases (denoted L1/L2 bias here) is also not a concern to the general GPS user. If the measured differential L1/L2 delay from a given satellite/receiver pair is used to remove the ionospheric effects from a GPS range measurement for that same pair by the usual two-frequency correction technique, then the SV and receiver L1/L2 biases will also be removed with no adverse effect.

Thus, these L1/L2 biases do not affect the large majority of GPS users. The only users who are affected by these differential biases are those who use the GPS ionospheric measurements to remove the ionospheric effects from other systems. For these applications the SV and receiver L1/L2 biases must be separated from the ionospheric delay.

Some examples of systems that are affected by this SV and receiver L1/L2 bias problem are NASA's Deep Space Network (DSN) calibration system, which plans to use GPS measurements for ionospheric calibration of the DSN and the Air Force's Transionospheric Sensing System (TISS), a world-wide network of GPS receivers which will generate ionospheric estimates for a variety of DoD systems. This problem could also affect the GPS geodetic user who attempts to use the ionospheric measurement from one receiver to calibrate a different receiver.

The SV L1/L2 biases for the current constellation of Block I GPS SVs are somewhat problematic. A recent study by Lanyi (1986) suggests that the pre-launch calibration values (Winn 1988) of the SV L1/L2 biases do not agree with the current values estimated from satellite-to-ground transmissions. The magnitude of the SV L1/L2 biases reported from both of these sources generally range from 0 to 6 ns of differential delay but an SV-by-SV comparison shows discrepancies as large as 4 ns. One of the main purposes of this study was to investigate the differences between the SV L1/L2 biases reported from these two sources.

The L1/L2 biases contributed by GPS receivers have not been widely investigated because these biases are not important to the general GPS user. The manufacturer of the GPS receiver may state that the receiver L1/L2 bias is negligible (to the general geodetic user), but usually does not give a specific value for the calibration nor its level of accuracy (Ward 1982; Texas Instruments 1982).

The accuracy required for ionospheric measurements varies from system to system. Many systems require an accuracy of 1 ns or less, and so both the SV and receiver L1/L2 biases would have to be estimated to at least this level of accuracy. This is not currently possible due to the lack of information about the SV and receiver L1/L2 biases.

In this study ARL:UT has tried to address four specific questions related to the SV L1/L2 biases.

- (1) What are the SV L1/L2 biases measured from SV-to-receiver transmissions?
- (2) How do these measured SV L1/L2 biases compare with the pre-launch calibration values?
- (3) Do the SV L1/L2 biases show any systematic variation over an extended period of observation?
- (4) What are the accuracy limits of the SV L1/L2 bias estimates?



The relationship between GPS measurements and ionospheric TEC is discussed in Section 2. The technique used to estimate the SV plus receiver L1/L2 biases is described in Section 3. Section 4 contains information about the data used to estimate the biases and Section 5 contains the results from the estimation process. Section 6 compares the SV L1/L2 biases estimated from this study with SV L1/L2 biases from other sources. A discussion of the accuracy of the ionospheric model used in the estimation process is given in Section 7. The results of the investigation of the receiver L1/L2 biases are presented in Section 8. In Section 9, future improvements are discussed, and a summary of the results is presented in Section 10.

## 2. IONOSPHERE TOTAL ELECTRON CONTENT MEASUREMENTS USING GPS

The GPS satellites are the sources of signals used in this study to obtain the differential group delay measurements. These satellites have been placed in orbit approximately 22,000 km above the earth and transmit coherent radio signals at two L-band frequencies, L1 at 1575.42 MHz and L2 at 1227.6 MHz. The satellite transmit time is encoded in two separate codes: the coarse acquisition (C/A) code and the precise (P) code. The C/A code is normally present only on L1, and the P code is present on both L1 and L2. In this study we have used data exclusively from receivers that track the more accurate P code.

As L1 and L2 signals travel through the ionosphere they are delayed by different amounts due to the dispersive nature of the ionosphere. When the two signals are received by the ground station they are time tagged and their satellite-to-receiver transit times (received time minus transmit time) are recorded. The difference between the L1 and L2 transit times is the differential group delay and is related to the TEC by

$$TEC = K \cdot T_{dd} \cdot \frac{1}{1 - (f_1/f_2)^2} \quad (2.1)$$

$$T_{dd} = T_2 - T_1 \quad (2.2)$$

where

$T_{dd}$  is the differential group delay between L1 and L2 (ns),

$T_1$  is the transit time of the L1 signal,

$T_2$  is the transit time of the L2 signal,

TEC is the total electron content (el/m<sup>2</sup>),

K is the constant  $-1.85 \times 10^{16}$  (el/m<sup>2</sup>s),

$f_1$  is the frequency of L1, and

$f_2$  is the frequency of L2.

This can be expressed in more convenient units of total electron content units (TECU), where one TECU is equivalent to  $10^{16}$  electrons/meter<sup>2</sup> (el/m<sup>2</sup>). Then,

$$\text{TEC (TECU)} = 2.85 \cdot T_{\text{dd}} \text{ (ns)} \quad (2.3)$$

The TEC value derived from the differential group delay is a measure of the number of electrons along the satellite-to-station line-of-sight.

The ionosphere also advances the phases of the L1 and L2 carrier waves by different amounts. As the signals are received at the ground station their phase changes over time are recorded. Dividing the phase change by the frequency of the signal converts the phase change to units of time. This scales the ionospheric effects on the two frequencies to a common base. The difference between the phase change of L1 and L2 in units of time is referred to as the differential carrier phase advance. These measurements are more accurate than the group delay measurements, but these measurements contain an unknown bias due to the integer cycle ambiguity of phase measurements. This integer cycle ambiguity exists because the number of full phase cycles between the satellite and the receiver is unknown.

The phase advance measurements are more accurate than group delay measurements and are also less susceptible to multipath effects. The TI 4100 GPS receiver has an internal measurement noise for differential carrier phase advance at the 5 ps level, whereas the corresponding noise for differential group delay is at the 1 ns level. Multipath noise is strongly dependent on the local antenna environment but rough estimates derived from the ARL:UT rooftop show that multipath contributes about 10 ps to differential carrier phase advance measurements and 2 ns to differential group delay. The difference in the noise contributions of these measurements may be explained by comparing the characteristic lengths of the signals from which the measurements are obtained.

The fundamental clock rate of the GPS system is 10.23 MHz ( $f_0$ ). This frequency is also the bit rate of the P code, while the C/A code bit rate is one tenth of this. The actual GPS carriers are at  $154 f_0$  (L1) and  $120 f_0$  (L2). The P code therefore has an effective characteristic length of about 30 m, which is larger than the L1 and L2 carrier wavelengths by factors of 154 and 120. The difference in the P code characteristic lengths and the carrier wavelengths is

the underlying reason for the large differences in the receiver noise and multipath contributions for the two measurement types. The actual resolution of these measurements can be improved by a factor of about 100 by using appropriate interpolation techniques, but the measurement accuracy ratio between the measurement types will remain the same.

GPS group delay measurements contain biases from a variety of sources. Those sources which generate biases which have the same value for L1 and L2 do not affect the differential group delay measurement. These sources include SV and receiver clock biases, tropospheric delays, orbit errors, and other frequency independent delays. Those biases common to both frequencies are removed when the L1 and L2 measurements are differenced. The differential L1/L2 biases are generated by the different circuitry paths that the signals travel in the SV and receiver hardware.

### 3. LEAST SQUARES ESTIMATION OF L1/L2 BIASES

A least squares estimation technique is used to estimate the combined satellite plus receiver L1/L2 biases and a set of ionospheric model parameters from the TI 4100 measurements. This technique fits the differential group delays measured over an observation span of about 5 hours to a model which includes both the satellite and receiver L1/L2 biases and parameters of an ionospheric TEC model. The ionospheric TEC model is a second-order polynomial in an earth-centered frame which co-rotates with the sun. This is effectively a local time/latitude co-rotating frame. The use of this TEC model assumes that the TEC is independent of time in the co-rotating reference frame over the duration of the observation session. This model is local, that is, it is applied over only a small region of the world.

This least squares estimation technique is similar to the one used by Lanyi (1986). One major difference is that Lanyi used SERIES receivers which collect only group delay data while ARL:UT used TI 4100 receivers which collect both group delay data and carrier phase advance data. The inclusion of the carrier phase in the estimation process should reduce the noise contributions and provide a more accurate estimate of the SV L1/L2 biases.

The GPS receiver differential group delay measurements include the differential delays due to the SV, the receiver, and the ionosphere, expressed as:

$$T_m = R + S + I \quad , \quad (3.1)$$

where

$T_m$  is the differential delay measured by the receiver,

$R$  is the differential delay caused by the receiver (receiver L1/L2 bias),

$S$  is the differential delay caused by the satellite (SV L1/L2 bias), and

$I$  is the differential delay caused by the ionosphere.

In general, none of the differential delays  $R$ ,  $S$ , or  $I$  are known, although  $R$  and  $S$  could possibly be measured through laboratory calibration.

The SV and receiver L1/L2 biases are assumed to be constant throughout the observation session, but the differential delay caused by the ionosphere changes. In the co-rotating reference frame, this change is mainly due to the slowly varying satellite-to-receiver elevation angle. This elevation angle dependence allows one to separate the varying ionosphere delay from the constant SV and receiver L1/L2 biases. The SV and receiver biases, however, cannot be estimated independently of each other using this technique alone. If all of the measurements are from a single receiver, then a set of combined SV plus receiver biases can be estimated. These combined biases will have a different value for each SV observed, and the receiver bias contribution will be a constant value throughout the set. These combined biases are referred to as SV plus receiver (SPR) L1/L2 biases in this report.

Several assumptions are required to develop the ionospheric model used in the estimation technique. First, it is assumed that the TEC variations in the ionosphere are due only to solar effects over the time span of interest. This means that the ionosphere is assumed to be constant in a frame which co-rotates about the earth with the sun. This allows us to ignore the time variation of the ionosphere. Second, it is assumed that the variations of the ionospheric TEC can be modeled with a low order polynomial model. This assumption is valid over only a limited geographic extent and it is most valid during times when the ionosphere is quiet, such as nighttime. It may not be feasible, however, to use this assumption with data taken during periods when the TEC is changing rapidly, such as dawn and dusk. The third assumption required by the model is that the SPR L1/L2 biases remain constant over the time span modeled.

In order to incorporate the one-dimensional (station-to-satellite) slant TEC measurements into the two-dimensional (latitude, longitude) ionospheric model, two steps must be taken. First, the slant TEC measurements must be converted to equivalent vertical TEC values and second, these vertical TEC values must be assigned to a point in the two-dimensional model. The implications of these two required steps are discussed in more detail in Appendix B, but we should mention here that these steps are very closely related to the problem of determining the M factor, which is required to utilize Faraday rotation measurements for measuring TEC (Titheridge 1972). These

steps introduce some degree of error due to the non-uniformity of the ionosphere, but the errors are rather small when a smooth nighttime mid-latitude ionosphere is under consideration.

The first step is achieved by using a standard obliquity factor model, which is described in the Appendix and is shown in Fig. 3.1. This obliquity factor,  $Q$ , converts the slant TEC to an equivalent vertical TEC. The second step is achieved by assigning the vertical TEC derived from the measured TEC to the point where the line of sight intersects the ionosphere at 350 km in the co-rotating reference frame. This point is referred to as the ionospheric intercept point and is illustrated in Fig. 3.2.

The second order polynomial used to model the ionospheric vertical TEC is given by

$$M(\phi, \lambda_{cr}) = c_1 + c_2 \cdot \phi + c_3 \cdot \lambda_{cr} + c_4 \cdot \phi^2 + c_5 \cdot \lambda_{cr}^2 + c_6 \cdot \phi \cdot \lambda_{cr} \quad , \quad (3.2)$$

where

$M$  is the vertical TEC,

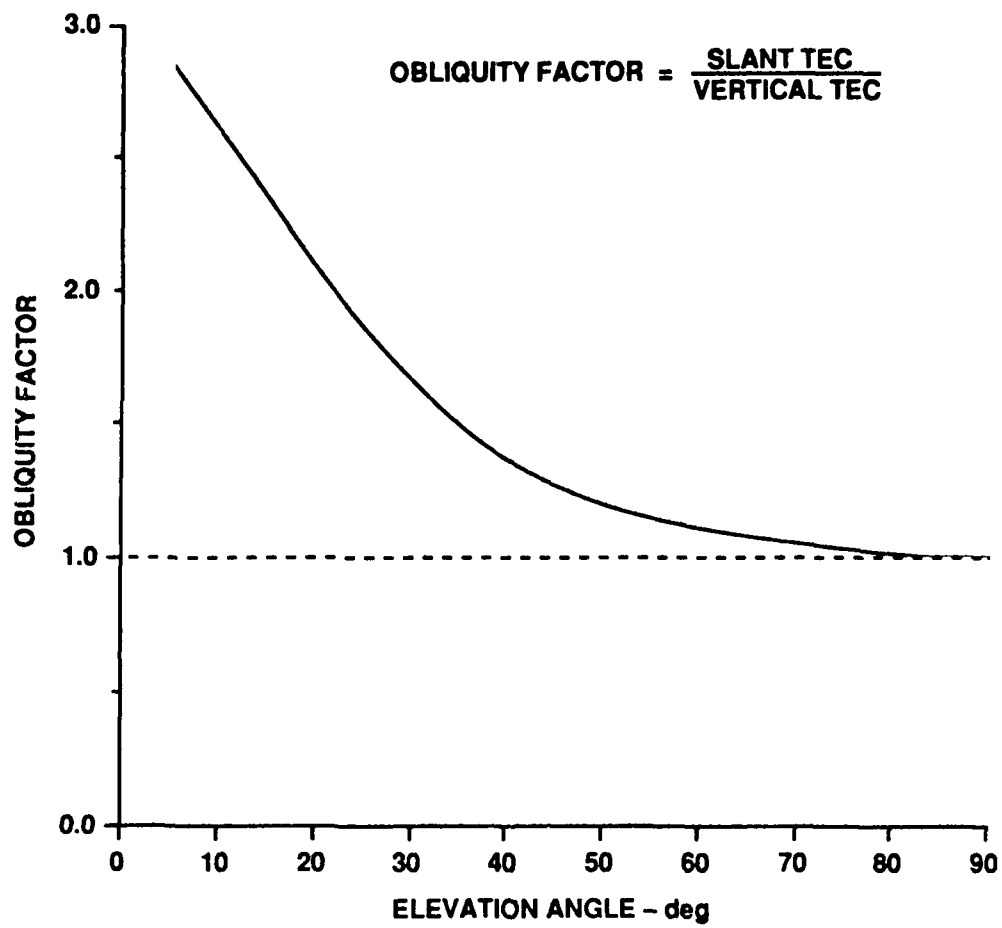
$\phi$  is the latitude of the ionospheric intercept point,

$\lambda_{cr}$  is the longitude of the ionospheric intercept point in the co-rotating reference frame, and

$c_1, \dots, c_6$  are the ionosphere model coefficients.

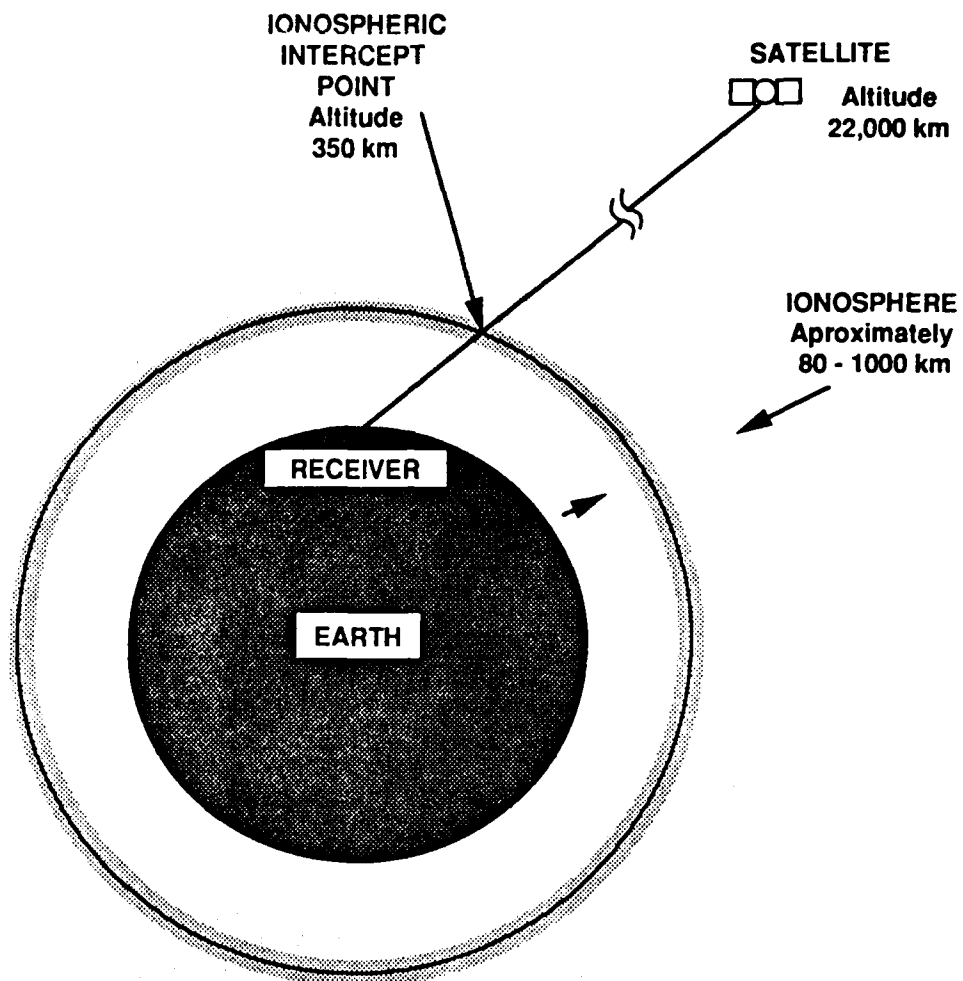
The longitude of the ionospheric intercept point in the co-rotating reference frame depends on both the ionospheric intercept longitude in an earth fixed reference frame and the rotation of this earth fixed frame relative to the sun:

$$\lambda_{cr} = \lambda_{ef} + T_e \cdot \omega_e \quad , \quad (3.3)$$



**FIGURE 3.1**  
**OBLIQUITY FACTOR MODEL**





**FIGURE 3.2**  
**ILLUSTRATION OF IONOSPHERIC INTERCEPT POINT GEOMETRY**

where

$\lambda_{cr}$  is the longitude of the ionospheric intercept point in the co-rotating reference frame,

$\lambda_{ef}$  is the longitude of the ionospheric point in an earth fixed reference frame,

$T_e$  is the time of day (UT), and

$\omega_e$  is the angular velocity of the earth.

The co-rotating reference frame in this study was chosen such that the sun's longitude is  $180^\circ$ . The latitude is not affected by the earth's rotation, and is therefore identical to the ionospheric intercept latitude in the earth fixed reference frame. This co-rotating reference frame is adequate for modeling a mid-latitude ionosphere, but a more sophisticated reference frame, which takes into account the earth's tilt and the differences of the geographic and geomagnetic coordinate frames, may be required for other regions.

If the data are taken by a single receiver, then the ionospheric delay measurements can be described by

$$D_{ij} = B_j + Q(E) \cdot M(\phi, \lambda_{cr}) \quad , \quad (3.4)$$

where

$D_{ij}$  is the measurement of ionospheric delay,

$i$  is the time tag index,

$j$  is the satellite index,

$B_j$  is the SPR L1/L2 bias,

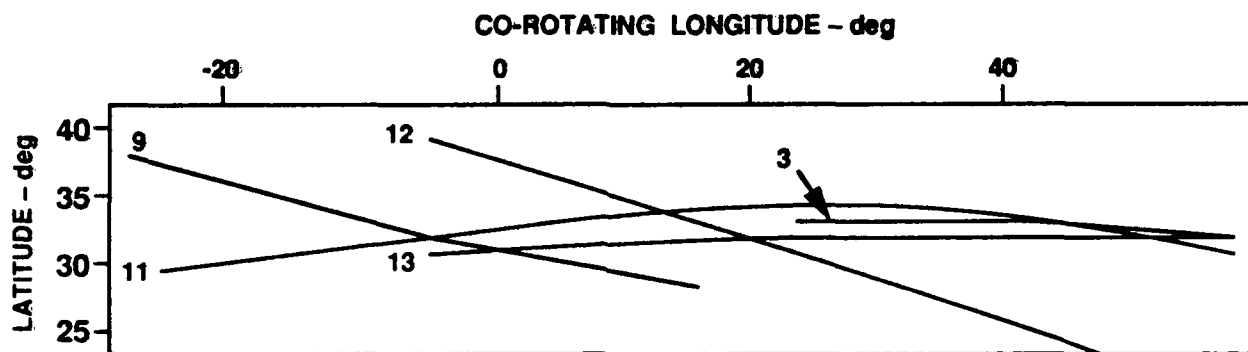
$Q(E)$  is the obliquity factor as a function of elevation angle  $E$ , and

$M(\phi, \lambda_{cr})$  is the vertical TEC model of the ionosphere as a function of ionospheric intercept latitude and longitude in the co-rotating reference frame.

In this equation, the elevation angle, ionospheric intercept latitude, and longitude are known from the ground station position and satellite ephemerides. The SPR L1/L2 bias parameters  $B_1$ - $B_n$ , where  $n$  is the number of satellites being observed, and the ionosphere model parameters  $c_1$ - $c_6$  are estimated

using a least squares estimation technique. If all the data are taken from a single hardware channel of a given receiver, the receiver L1/L2 bias is the same for each estimated SPR L1/L2 bias; therefore, differencing the estimated SPR L1/L2 biases between SVs will give the differences in the SV L1/L2 biases.

During a typical 5 hour observation session, only a small portion of the total ionosphere is modeled because the ionospheric intercept points cover only a relatively small region as shown in Fig. 3.3. This figure shows the ionospheric intercept paths for an actual observation session at Austin, Texas. The numbers on the paths are the SV PRN identification numbers. For a station latitude of  $30^\circ$ , ionospheric intercept points range over approximately  $15^\circ$  of latitude and  $90^\circ$  longitude in the co-rotating reference frame.



STATION: AUSTIN, TX  
 DAY 79, 1987  
 11 P.M. TO 4 A.M. LOCAL TIME (CST)

FIGURE 3.3  
 IONOSPHERIC INTERCEPT TRACKS

#### 4. DESCRIPTION OF THE DATABASE - CETKE I CAMPAIGN

In selecting a database to be used for the least squares estimation process for SPR L1/L2 biases several characteristics were required. Differential group delay and phase advance measurements gathered over several weeks were desirable to allow an investigation of the variation of the SPR L1/L2 biases over time. The data collection scenario had to be designed to provide adequate coverage of the ionospheric region to be modeled. Nighttime data were greatly preferred to give a quiet ionosphere which could be more easily modeled. A single hardware channel receiver, such as the TI 4100, was preferred so that only a single receiver L1/L2 bias would have to be considered for each receiver. GPS receivers with multiple hardware channels usually have a method for calibrating inter-channel biases, but this would complicate the estimation process. Data from multiple ground stations, with overlap in the ionospheric area sampled, were desired to allow comparisons of ionospheric models derived from different stations for the same time span.

A database was available which met all of the major requirements of the experiment; therefore, a data collection campaign was not required. The data chosen to be used for this study were taken during the Clock Evaluation and Timekeeping Experiment (CETKE I) campaign, a joint effort of ARL:UT, National Bureau of Standards (NBS), and the United States Naval Observatory (USNO). Approximately five weeks of data were taken on 18 March - 24 April 1987 at three locations: Austin, Texas; Boulder, Colorado; and Washington, D.C. (see Fig. 4.1). The data were recorded using TI 4100 receivers which used the same scenario each day. The five satellites used from the CETKE I data were PRNs 3, 9, 11, 12, and 13.

The data provide both group delay and carrier phase advance measurements at a nominal 60 s data rate with some periods of 6 s data. To obtain a more accurate estimate of the TEC, phase advance measurements were used to smooth the group delay measurements. The phase advance measurements provide the change in TEC relative to the beginning of the pass with a very high accuracy, while the differential group delay provides an estimate of the offset for the absolute TEC. This new set of measurements with

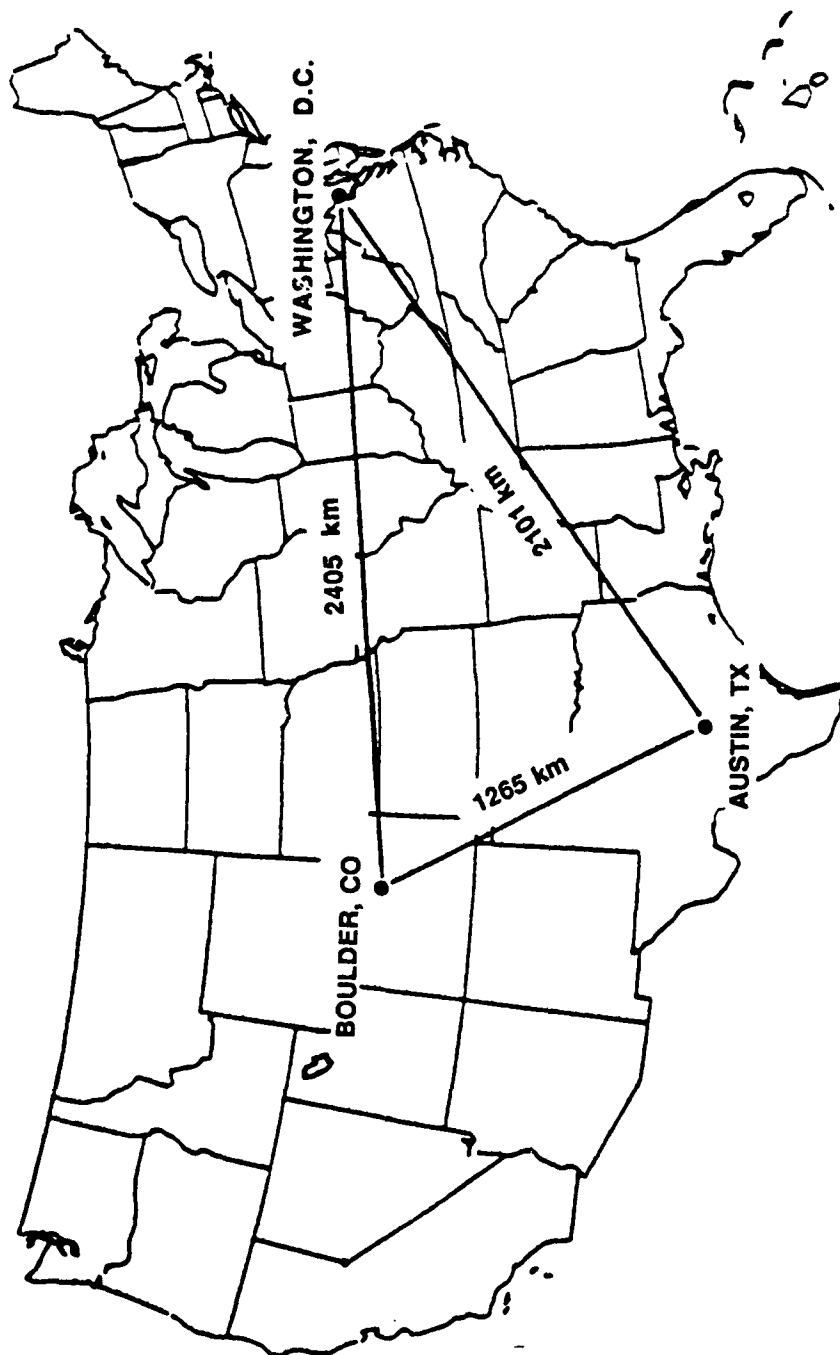
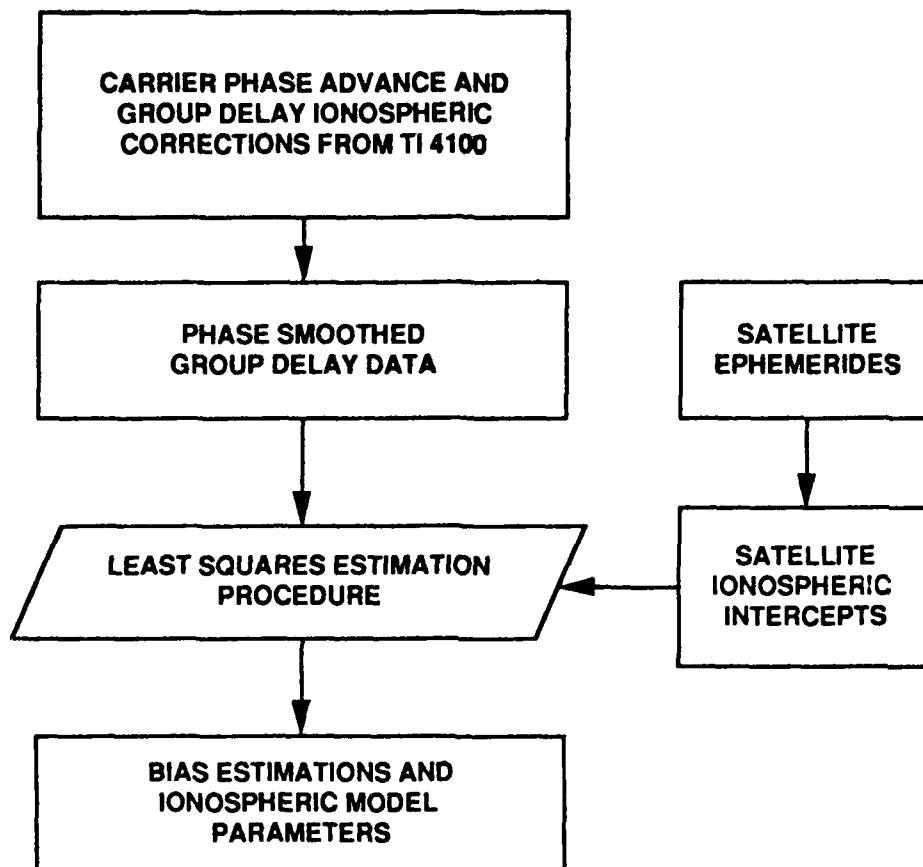


FIGURE 4.1  
CLOCK EVALUATION AND TIMEKEEPING EXPERIMENT (CETKE 1)  
GROUND STATIONS

group delay measurements smoothed by phase advance measurements is referred to as phase smoothed measurements. This smoothing process is discussed in more detail in Appendix A. Figure 4.2 shows the data processing flow for the estimation process.

The data collection time span must be long enough to provide a sufficient number of data points for the estimation process, but it must also be short enough that time dependent variations of the ionosphere are small. The data must also span a wide range of elevation angles so that the obliquity factor will have sufficient variation to permit separation of the SV and receiver biases from the ionosphere model. To fulfill these requirements nighttime data taken from local 11 p.m. to 4 a.m. (CST) were used. This time frame has a reasonable amount of sky coverage and is also the time span when the local ionosphere is usually quietest.



**FIGURE 4.2**  
**IONOSPHERIC DATA PROCESSING FLOW**



## 5. SPR L1/L2 BIASES ESTIMATED FROM THE CETKE I CAMPAIGN

The least squares estimation procedure was performed using data from each of the three stations over the five week period. For each station-day, estimates of the ionospheric model parameters and the SPR L1/L2 biases for five SVs were generated. The estimation procedure also calculated the formal error of each ionospheric parameter and SPR L1/L2 bias. The formal error indicates how closely the data fit the estimated parameter set for that day. The formal errors for all estimated SPR L1/L2 biases are quite similar for a given day but there is a significant day-to-day variation. These errors range from 0.02-0.18 ns over all the data.

SPR L1/L2 biases derived from the Austin station data are plotted in Fig. 5.1 in units of nanoseconds differential delay (ns dd). The standard deviation of the daily SPR L1/L2 biases for each SV over the five week period is also shown in this figure. The average standard deviation over all SVs is 0.5 ns. There does not appear to be a general trend of the biases for any of the SVs. Over this time period, however, there is an apparent correlation of the day-to-day changes in the SPR L1/L2 biases across all SVs. This correlation is probably due to a mismodeling of the ionosphere, which would be common to all SVs.

This correlated error can be removed by subtracting the mean (over all SVs) SPR L1/L2 bias from each of the estimated biases for that day. Figure 5.2 is a plot of the biases after the mean daily biases have been removed. Removing this correlated error reduces the average standard deviation across all SVs to about 0.3 ns. This gives a fair estimate of the accuracy limits this type of estimation could provide with an improved ionospheric model.

Statistics of the daily SPR L1/L2 biases estimated for each station are plotted in Fig. 5.3. The SPR L1/L2 biases were estimated for each day Fig. 5.4 over a five week period for the five SVs using data from each of the three stations separately. Each plot in Fig. 5.3 contains the mean, standard deviation, and minimum/maximum values over the five week period for the estimated SPR L1/L2 biases for the individual stations. Table 5.1 contains the average of the

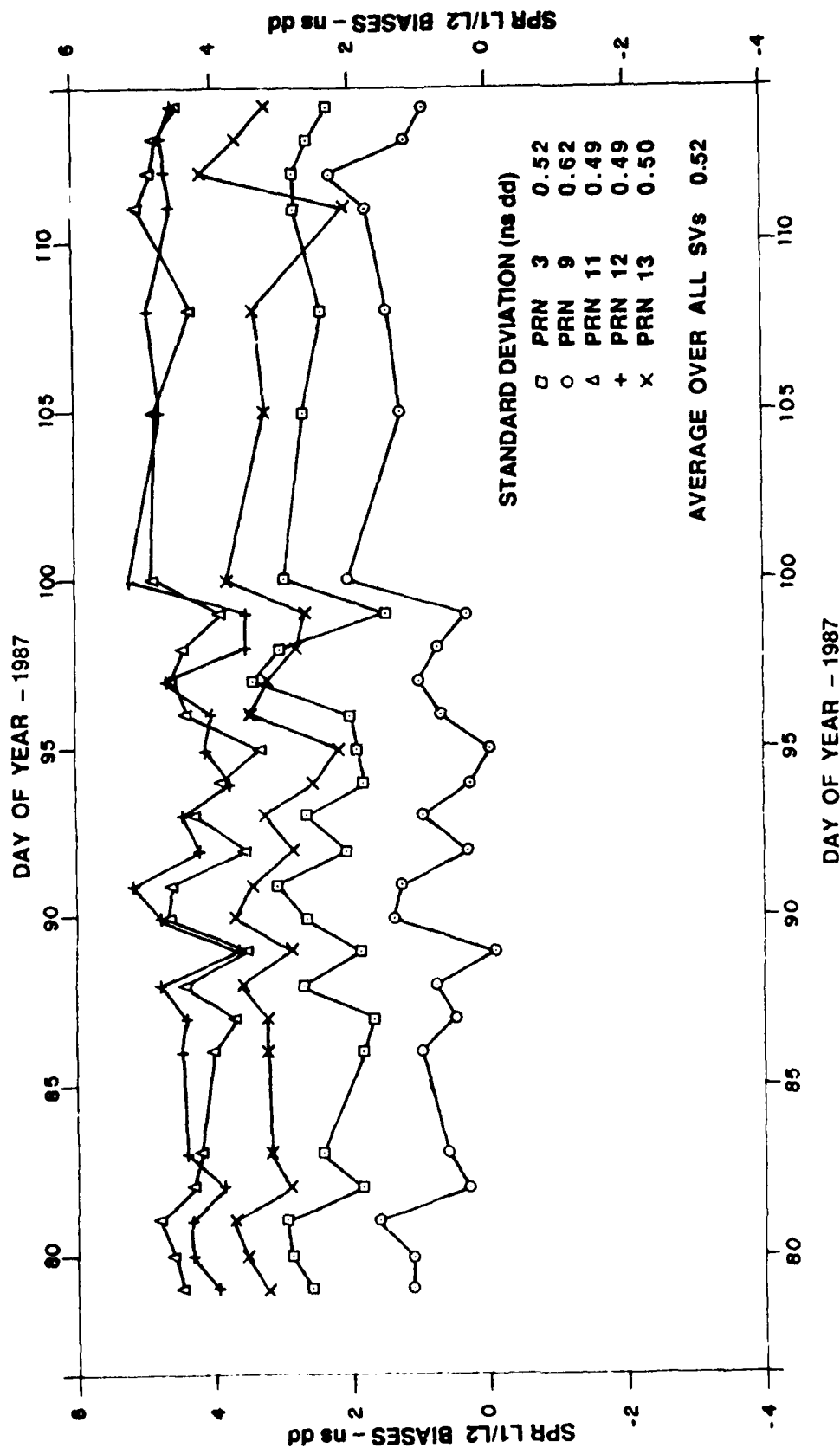


FIGURE 5.1  
SPR L1/L2 BIASES - AUSTIN, TEXAS

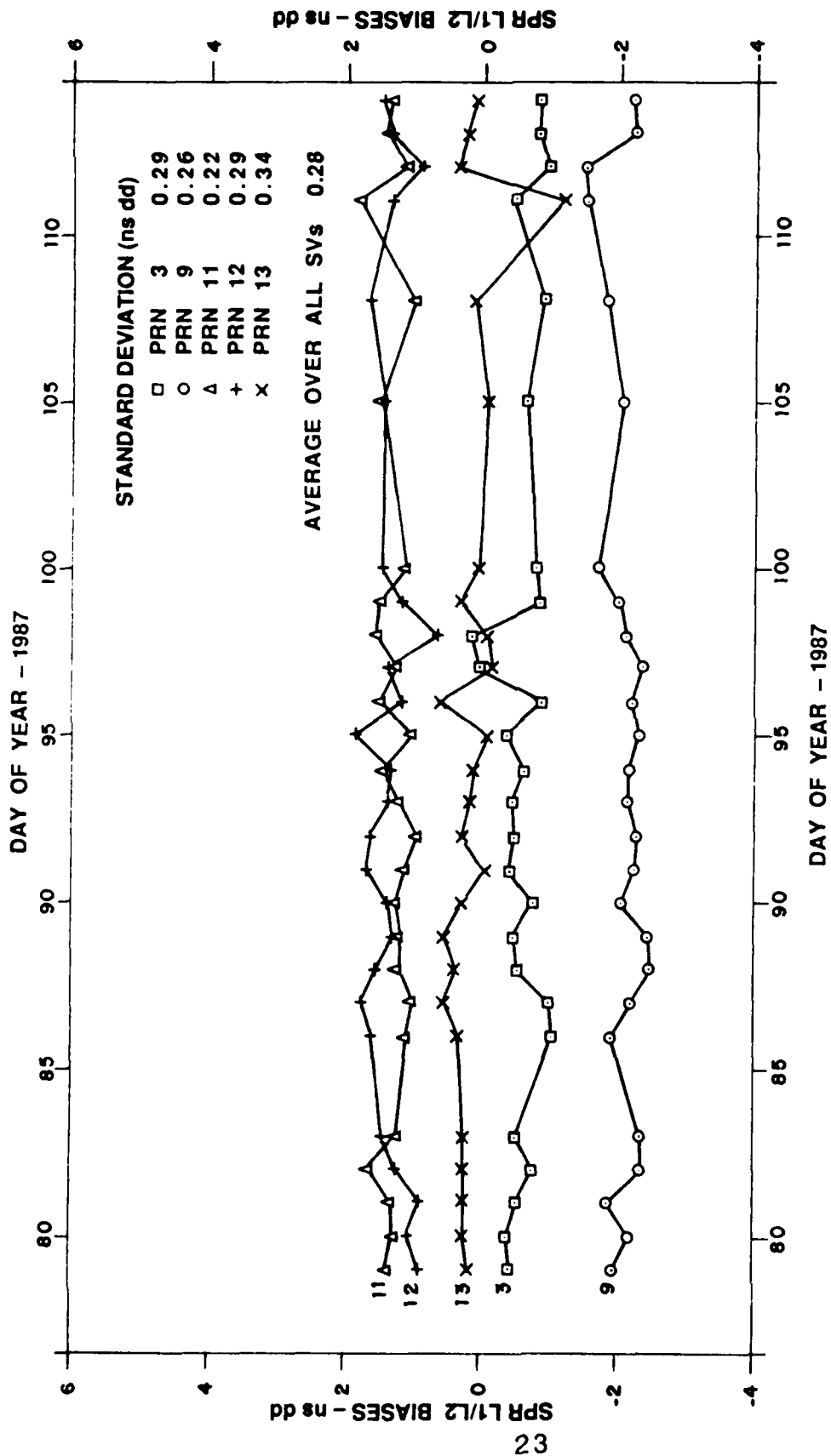


FIGURE 5.2  
SPR L1/L2 BIASES RELATIVE TO DAILY MEANS - AUSTIN, TEXAS

DAY 77 TO 114 - 1987

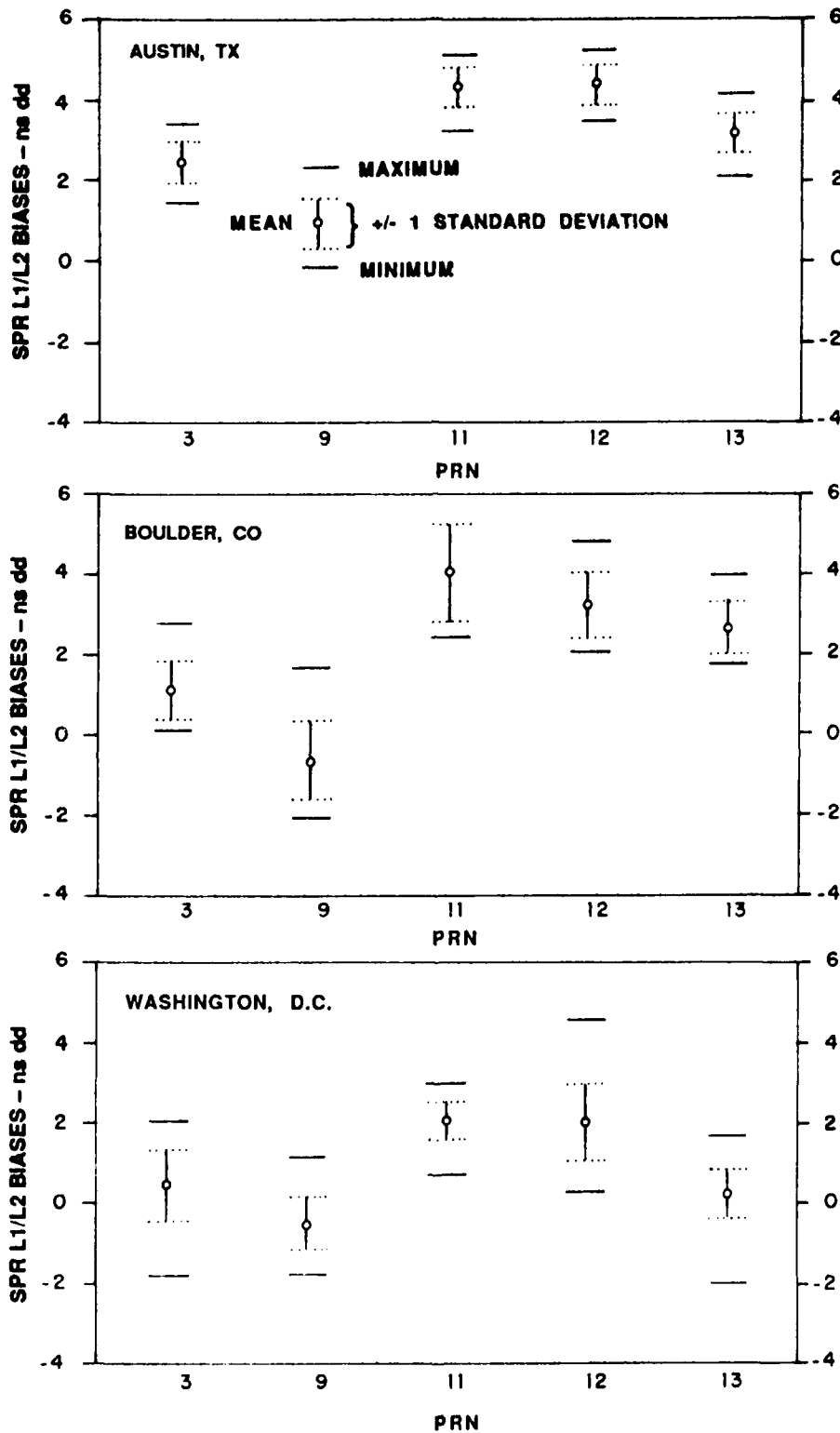


FIGURE 5.3  
SPR L1/L2 BIAS ESTIMATION STATISTICS  
FOR A 5 WEEK PERIOD

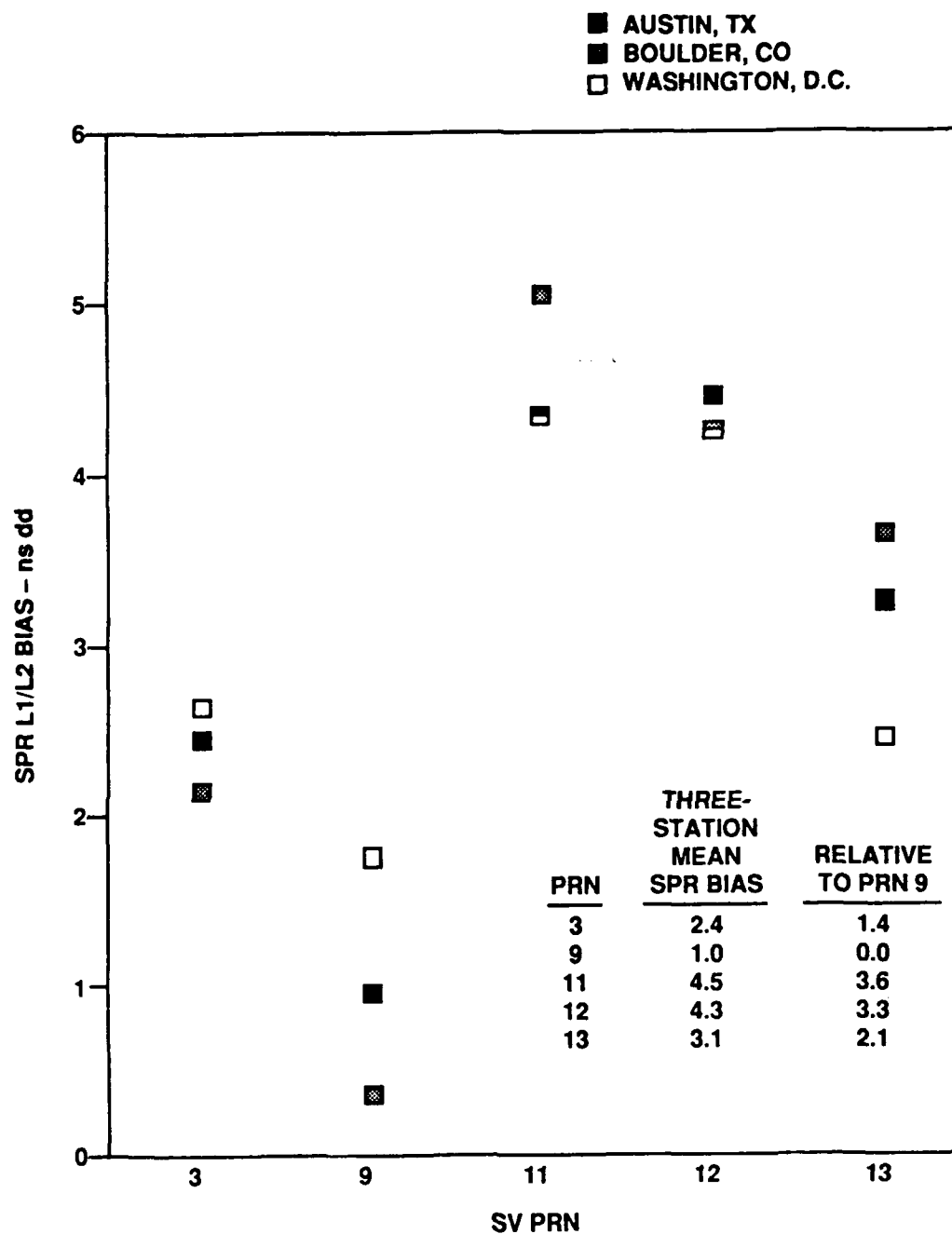


FIGURE 5.4  
 SPR L1/L2 BIASES - ADJUSTED TO AUSTIN'S RECEIVER L1/L2 BIAS

TABLE 5.1  
ESTIMATED SPR L1/L2 BIASES -  
AVERAGES OVER A FIVE WEEK PERIOD FROM CETKE I DATA  
(ns of Differential Delay)

STATION	SV PRN				
	3	9	11	12	13
AUSTIN	2.4	0.9	4.3	4.4	3.2
BOULDER	1.1	-0.7	4.0	3.2	2.6
WASHINGTON, D.C.	0.4	-0.5	2.1	2.0	0.2

SPR L1/L2 biases over the five week period for each of the three stations. This table indicates that there are offsets between average SPR L1/L2 biases from the three stations. The differences in receiver biases are the most probable sources of these offsets, although the ground station's multipath could also be a contributing factor.

To estimate the L1/L2 bias differences between receivers, the mean of the SPR L1/L2 biases for each station is differenced. As shown in Table 5.2, the receiver differences with respect to Austin are 1.0 ns for Boulder and 2.2 ns for Washington, D.C. These values are not estimates of the absolute receiver L1/L2 biases, but are only the receivers' L1/L2 biases relative to another receiver.

To remove differences due to the receivers' L1/L2 biases, 1.0 ns is added to the Boulder SPR L1/L2 bias estimates and 2.2 ns is added to the Washington SPR L1/L2 bias estimates. Estimated SPR L1/L2 biases with Boulder and Washington, D.C., data adjusted to Austin's receiver L1/L2 bias are shown in Fig. 5.4. Also listed on this figure are the mean SPR L1/L2 biases for each SV and the difference in SPR L1/L2 biases relative to PRN 9. These are differences in SV L1/L2 biases only since the common receiver L1/L2 bias has been removed by the differencing.

TABLE 5.2  
ESTIMATE OF RECEIVER L1/L2 BIAS DIFFERENCES - CETKE I  
(ns of Differential Delay)

STATIONS DIFFERENCED	SV PRN					ALL SVs	
	3	9	11	12	13	MEAN	STANDARD DEVIATION
BOULDER-AUSTIN	-1.3	-1.6	-0.3	-1.2	-0.6	-1.0	0.5
WASHINGTON, D.C. - AUSTIN	-2.0	-1.4	-2.3	-2.4	-3.0	-2.2	0.6



## 6. COMPARISON OF ESTIMATED SPR L1/L2 BIASES WITH OTHER SOURCES

The GPS SV L1/L2 biases were measured by Rockwell International in pre-launch factory tests (Winn 1988), but apparently have not been officially checked since then. The Rockwell test results are given in Table 6.1. These are given with the sign for L2-L1.

The pre-launch SV L1/L2 biases, the SPR L1/L2 biases from this study (labeled as ARL) and the SPR biases from Lanyi (1986) (labeled as JPL) are listed in Table 6.2. The ARL values are those listed in Fig. 5.3 of this report and the JPL values are obtained from Fig. 5.4 (Day 22, 1984) (Lanyi 1986). The biases in this table from the different sources cannot be directly compared because both the ARL and JPL values contain unknown receiver biases. The inter-satellite bias differences, however, can be compared between the three sources because this differencing removes the receiver bias effects. These inter-satellite bias differences, relative to PRN 9, are listed on the left hand side of Table 6.3.

The differences between the three sources are shown on the right hand side of Table 6.3. This table shows that the ARL values are all within 1.0 ns of the pre-launch values for the four common SV pairs with an average difference magnitude of 0.5 ns. Each individual ARL/pre-launch difference is within the corresponding one sigma variation of the SV differences derived from the pre-launch factory test standard deviations. The one sigma variation of the SV differences is equal to the square root of the sum of the squares of the sigmas of the SVs in that pair. It is interesting to note that the two SVs with large ARL/pre-launch differences (PRNs 3 and 12) also have relatively large pre-launch standard deviation values.

The JPL/pre-launch differences vary from 0.0 to -4.1 ns for four common SV pairs with an average difference magnitude of 1.7 ns. Two of the individual JPL/pre-launch differences are within the corresponding one sigma variations

**TABLE 6.1**  
**L1/L2 SV BIASES FROM ROCKWELL INTERNATIONAL**  
**PRE-LAUNCH FACTORY TESTS**  
 (ns of Differential Delay, L2/L1)

<b>SV PRN</b>	<b>MEAN</b>	<b>STANDARD DEVIATION</b>
3	1.99	1.02
4	1.03	1.50
5	-0.68	0.63
6	2.33	0.94
8	1.53	1.14
9	-0.34	0.24
11	3.09	0.83
12	1.96	1.75
13	1.79	0.33

**TABLE 6.2**  
**COMPARISON OF SPR L1/L2 BIASES**  
 (ns of Differential Delay)

<b>PRN</b>	<b>SPR L1/L2 BIAS SOURCES</b>		
	<b>ARL</b>	<b>JPL</b>	<b>PRE-LAUNCH</b>
3	2.4	---	2.0
4	---	26.1	1.0
6	---	23.3	2.3
8	---	25.5	1.5
9	1.0	24.8	-0.3
11	4.5	26.6	3.1
12	4.3	---	2.0
13	3.1	---	1.8

**TABLE 6.3**  
**COMPARISON OF SPR L1/L2 BIASES**  
**RELATIVE TO PRN 9**  
**(ns of Differential Delay)**

PRN	SPR L1/L2 BIAS SOURCE PRE- LAUNCH (PL)			SPR L1/L2 BIAS DIFFERENCES		
	ARL	JPL	LAUNCH (PL)	ARL/JPL	ARL/PL	PL/PL
3-9	1.4	---	2.3	---	-0.9	---
4-9	---	1.3	1.3	---	---	0.0
6-9	---	-1.5	2.6	---	---	-4.1
8-9	---	0.7	1.8	---	---	-1.1
11-9	3.5	1.8	3.4	1.7	0.1	-1.6
12-9	3.3	---	2.3	---	1.0	---
13-9	2.1	---	2.1	---	0.0	---

derived from the pre-launch factory test, while the other two are outside the one sigma variations. An extensive SV-by-SV comparison between the ARL and JPL estimates is not possible because only PRNs 9 and 11 were used by both. The ARL/JPL difference for this pair is 1.7 ns. It should be noted that more recent unpublished SPR L1/L2 biases from JPL using the new ROGUE receiver appear to be in closer agreement with both ARL and pre-launch values (Lanyi 1988). The ROGUE receiver collects both group delay and carrier phase advance measurements.

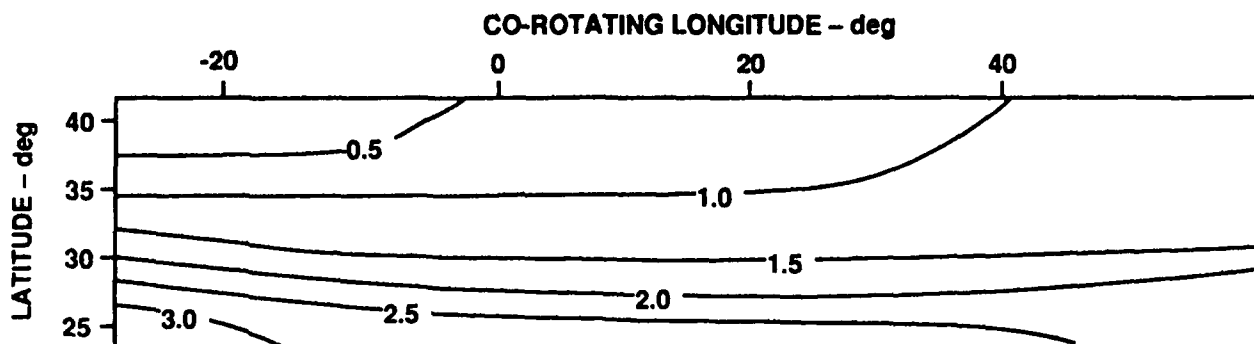
To ensure that the differences in the SPR L1/L2 bias estimates from Lanyi (1986) and the current study are not due to differences in the estimation process itself, a common data set was processed through both the ARL:UT and JPL estimation programs. The results were found to be almost identical; the only differences resulted from slightly different data editing processes used by the different programs.

## 7. DISCUSSION OF THE IONOSPHERIC MODEL USED IN THE ESTIMATION TECHNIQUE

The ionospheric model is the key element in the estimation technique. If the model can adequately describe the variations of the ionosphere over the time and geographical boundaries of the data set, then the combined SV and receiver L1/L2 biases can be successfully estimated. A low order polynomial is likely to be adequate to estimate the TEC during a nighttime session in the mid-latitude regions, but daytime sessions or stations at equatorial or polar latitudes will require a more complex model. The complex ionospheric variations in these conditions will restrict the potential accuracy level of the estimated biases and ionospheric model parameters. This restriction is a strong argument for estimating the SV and receiver L1/L2 biases under conditions for which the ionosphere can easily be modeled rather than performing the estimation for each data set collected.

The preferred method of estimating the adequacy of the ionospheric model would be to compare the GPS derived ionospheric model against measurements from another source. No suitable ionospheric measurements from another source were available for the CETKE I campaign, so an alternative means of estimating the accuracy of the model was used. The ionospheric model derived from the measurements at one station was compared with the model derived from another.

A contour plot of the TEC values derived from the ionospheric model parameters estimated from one night of Austin data is shown in Fig. 7.1. There is only a slow variation of the TEC across the region during this session. The boundaries of applicability for this model are somewhat arbitrary, but certainly should depend upon the ionospheric intercept points (IIPs) corresponding to the measurements. The boundaries for the model shown in Fig. 7.1 were determined by using the maximum extent of the IIPs in latitude and longitude in the co-rotating reference frame to form constant latitude and longitude boundaries. The confidence in the model Fig. 7.1 values within these boundaries is greater in regions where the ionospheric intercept tracks are more dense than in the other regions.



STATION: AUSTIN, TX  
DAY 79, 1987  
11 P.M. TO 4 A.M. LOCAL TIME (CST)

FIGURE 7.1  
VERTICAL TEC FROM IONOSPHERE MODEL IN ns dd

ARL:UT  
AS-88-812  
DSC - GA  
8 - 1 - 88  
Rev 12-5-89

To compare the ionospheric model parameters derived from independent measurements at two stations, a common ionospheric model area was defined. This area was simply the intersection of the two boundary regions described above for two stations. The Austin sessions had very little or no common ionospheric viewing area when matched with either Boulder or Washington, but Boulder and Washington contained a substantial common viewing area due to the earth's rotation. The common viewing area for Boulder and Washington for UT Day 79, 1987, spanned  $11^\circ$  in latitude and  $60^\circ$  in co-rotating longitude. The model TEC contour plots for these two stations were quite smooth and had values ranging from 0.7 to 3.5 ns.

The model TEC values for these two stations were differenced at grid points spaced at  $1^\circ$  in latitude and  $5^\circ$  in longitude over the common viewing area. The average difference between these two models was 0.5 ns with a standard deviation of 0.6 ns. These differences are most likely due to a variety of sources, including inadequate sampling of the common viewing area (IIPs too sparse), inadequate modeling of the ionosphere, inadequacy of the assumption of a constant ionosphere in the co-rotating reference frame, and measurement errors. Although the contributions from all of these error sources can be reduced to some extent, this comparison gives a good estimate of the accuracies one can expect to obtain using GPS measurements to model a quiet nighttime mid-latitude ionosphere.

## 8. INTER-RECEIVER L1/L2 BIAS CALIBRATION

Comparison of the SPR L1/L2 biases from the three CETKE I stations indicates that the TI 4100 L1/L2 biases cannot be neglected in the overall calibration process. The results of the least squares estimation process show that the three CETKE I receivers' biases differ by 1.0 and 2.2 ns differential delay. It was suggested that these inter-receiver L1/L2 biases be checked by performing an inter-receiver calibration in a zero baseline type configuration so that the inter-receiver L1/L2 biases could be estimated independently of the SV biases. It has not yet been possible to perform this type of calibration for the three CETKE I receivers because the receivers have been used for other campaigns and have not been in a common location since the campaign. However, tentative plans are to calibrate at least two of the CETKE I receivers in the near future.

To demonstrate the type of results the inter-receiver calibration would provide and to gain a better understanding of the receiver bias differences, an inter-receiver calibration was performed on three other TI 4100 receivers, none of which was used in the CETKE I campaign. The inter-receiver calibration used the three receivers connected in a zero baseline configuration. In this configuration, the three receivers are connected to a common antenna/preamplifier and a common external clock. The receivers then collect data simultaneously. The differential group delay measurements contain delays due to the satellite, the ionosphere, and the receiver; but the satellite and ionosphere delays are the same for all receivers.

Using two of the receivers, the L1/L2 bias difference between them was estimated by first differencing L1 measurements between receivers and doing the same for the L2 measurements. The L1 and L2 inter-receiver differences were then differenced, resulting in L1/L2 receiver delay differences for this receiver pair. By this method of differencing, the satellite and ionospheric delays were removed, leaving only the differences in receiver L1/L2 biases.

The data used for this study spanned three days, with approximately 1.5 h of data for each day. Three days of data from three receivers, each of which is capable of tracking four satellites simultaneously, produced



12 receiver L1/L2 bias difference estimates for each receiver pair (a few less than this are present in the results because each receiver did not always track four satellites). The receivers used in this study are labeled as UT50, UT57, and UT58.

The noise in the L1/L2 bias differences is basically due to the pseudorange measurement noise of the TI 4100. The pseudorange measurement noise depends primarily on the bandwidths of the tracking loops and the signal strength. For L1/L2 difference measurements the important bandwidth is the delay lock difference bandwidth which was 0.5 Hz for the data collected for this inter-receiver calibration. The L1 pseudorange noise for this bandwidth has been measured in prior experiments (Coco and Clynch 1986) to range from 1 to 3 ns for typical signal strength levels; the L2 noise is slightly higher than this. The expected noise for the L1 - L2 measurement differenced between two receivers will be a factor of 2 higher than this L1 pseudorange noise, if we assume that the L1 noise is equal to the L2 noise. Thus, the expected noise will range from 2 to 6 ns for L1 - L2 receiver differenced measurements.

The receiver L1/L2 bias difference results are listed in Table 8.1. The standard deviation of individual data sets is about 3.0 ns (due to the receiver measurement noise of the group delay data), but the differences across days and trackers have only a standard deviation of about 0.3 ns. This means that although single data sets have large standard deviations, the mean bias differences are in close agreement across days and trackers. To check the consistency of the estimates the three-receiver closure given by

$$\langle \text{UT57} - \text{UT58} \rangle + \langle \text{UT58} - \text{UT50} \rangle = \langle \text{UT57} - \text{UT50} \rangle \quad (8.1)$$

is calculated from the average values given in Table 8.1:

$$\langle \text{UT57} - \text{UT58} \rangle + \langle \text{UT58} - \text{UT50} \rangle = -2.1 \text{ ns} \quad (8.2)$$

$$\langle \text{UT57} - \text{UT50} \rangle = -2.0 \text{ ns} \quad (8.3)$$

These values are very close, and therefore add confidence that the estimated values of receiver bias differences are correct.

TABLE 8.1  
INTER-RECEIVER L1/L2 BIAS DIFFERENCES - ZERO BASELINE TESTS

AVERAGE INTER-RECEIVER L1/L2 BIAS DIFFERENCE (ns dd) -  $\bar{X}$

RECEIVER		UT57-UT58			UT58-7T50			UT57-UT50		
PAIR: DATE:		3/17/87	3/18/87	3/19/87	3/17/87	3/18/87	3/19/87	3/17/87	3/18/87	3/19/87
TRACKER		----	-3.0	-2.6	1.0	0.7	0.8	----	-2.4	-1.7
1		-3.3	-3.5	-2.6	1.4	1.2	0.9	-2.0	-2.3	-1.8
2		-2.7	-2.8	----	0.8	0.6	0.6	-1.9	-2.2	-1.7
3		-2.9	-3.0	----	1.0	0.6	0.5	-2.1	-2.5	-1.9
4										

AVERAGE  
STANDARD  
DEVIATION

$$\langle \bar{X} \rangle = -2.9$$

$$\sigma_{\bar{X}} = 0.3$$

$$\langle \bar{X} \rangle = -0.8$$

$$\sigma_{\bar{X}} = 0.3$$

$$\langle \bar{X} \rangle = -2.0$$

$$\sigma_{\bar{X}} = 0.3$$

In the TI 4100 receiver the L1 and L2 frequency paths differ by only one L-band cavity filter path and a diplexer switch. Although the differential L1/L2 delay of the TI 4100 is said to be negligible (Ward 1982; Texas Instruments 1982) (for geodetic applications), the filter/diplexer is probably the source of the inter-receiver L1/L2 biases measured in the zero baseline tests.

## 9. SUGGESTIONS FOR IMPROVEMENTS

The combined SV plus receiver L1/L2 biases for five weeks of CETKE I campaign data have been estimated using a least squares estimation technique. These SV plus receiver (SPR) L1/L2 biases have a standard deviation of 0.5 ns differential delay over the five week period, but do not show a systematic variation over this time period. At each of the three ground stations, there is a day-to-day correlation in the SPR L1/L2 bias variations across the five SVs. These variations are probably due to a common mismodeling of the ionosphere common to all five SVs. When this correlation is removed, the standard deviation is reduced to 0.3 ns. This value represents an estimate of the SV L1/L2 bias accuracy level that can be achieved with an improved ionospheric model.

The receiver L1/L2 biases for the TI 4100 receivers used in the CETKE I campaign are not known at the level of accuracy of the SPR L1/L2 biases estimated in this study. Because the receiver L1/L2 biases are unknown, only the inter-satellite L1/L2 bias differences from this study can be compared to other sources. This receiver L1/L2 bias calibration problem is not restricted to the TI 4100; in fact, an independent L1/L2 calibration at this level of accuracy has not been demonstrated for any GPS receiver to date, to the best of our knowledge.

A number of issues need further study to improve the accuracy of the SV L1/L2 bias estimates. Different ionospheric models should be implemented to see if the accuracy of the SV L1/L2 biases can be increased. A third order polynomial model is the next logical step in complexity, but other types should be investigated. Another means of increasing the accuracy would be to increase the density of the ionospheric tracks in the modeled region. This could be achieved by using multiple receivers or by using a receiver that tracked more than four SVs. The question of how often the SV and receiver L1/L2 biases should be recalculated needs to be investigated. This could be done, to some extent, by searching for evidence of bias variations over a variety of time scales, varying from months to years.

## 10. SUMMARY

The inter-satellite differences of the estimated satellite plus receiver L1/L2 biases from the CETKE I campaign, the JPL SERIES receivers, and the pre-launch factory test values measured by Rockwell were compared. Only a limited number of SV pairs were compared between each source because data were not available for all SVs from each source. The four SV pairs common to CETKE I and the pre-launch tests differed by an average magnitude of 0.5 ns. The JPL values differed from the pre-launch values by larger amounts, with an average magnitude of 1.7 ns for four SV pairs. The single SV pair common to both CETKE I and JPL differed by 1.7 ns. In recent communications, Lanyi (1988) of JPL has indicated that results from the new ROGUE receiver show much better agreement with the pre-launch values than the SERIES results.

The ionospheric model is the key element in the estimation technique. A second order polynomial is used in the current study. To obtain an estimate of the accuracy of the ionospheric model, the vertical TEC values estimated from the model generated from one ground station (Boulder) were compared with those generated from a second station (Washington). The average difference between these two models was 0.5 ns with a standard deviation of 0.6 ns. This represents a rough measure of the accuracy of the ionospheric model generated from GPS measurements.

As mentioned above, the TI 4100 L1/L2 bias has not yet been calibrated absolutely. However, an inter-receiver L1/L2 bias calibration was performed using a zero baseline configuration with three TI 4100s connected to the same antenna and the same external clock. This calibration showed inter-receiver L1/L2 bias differences as large as 2.9 ns differential delay. These bias differences were constant over four trackers and three days of data. This confirms our assertion that the TI 4100 must be independently calibrated before it can be used to estimate absolute SV biases.

This study has compared the inter-satellite L1/L2 differences; the issue of the accuracy of absolute SV L1/L2 biases cannot be addressed until the receiver bias calibration problem has been resolved. This issue of absolute

calibration of SV L1/L2 biases is an important one because it would allow intercomparison of SV L1/L2 biases from different sources.

**APPENDIX A**  
**PHASE AVERAGING OF PSEUDORANGE MEASUREMENTS**

The carrier phase measurements of relative slant total electron are combined with the pseudorange measurements of absolute slant total electron content to generate a single observable. This observable, called the phase averaged pseudorange total electron content, is less noisy than the pseudorange measurements and is an absolute measurement, unlike the carrier phase measurements. The process of combining these two measurements is discussed below.

In the case of phase measurements, one has to deal with the fact that the received phase is known only up to a constant; phase is measured only up to a factor of  $2\pi$ . Once a signal has been acquired, the full cycles can be counted, but there will be an unknown number of cycles in the initial measurement. This is well known in the GPS geodetic community, where determining the "cycle ambiguity" is a key step in high precision differential positioning. In the ionospheric case, it means that an unknown bias remains in the phase based ionospheric measurements.

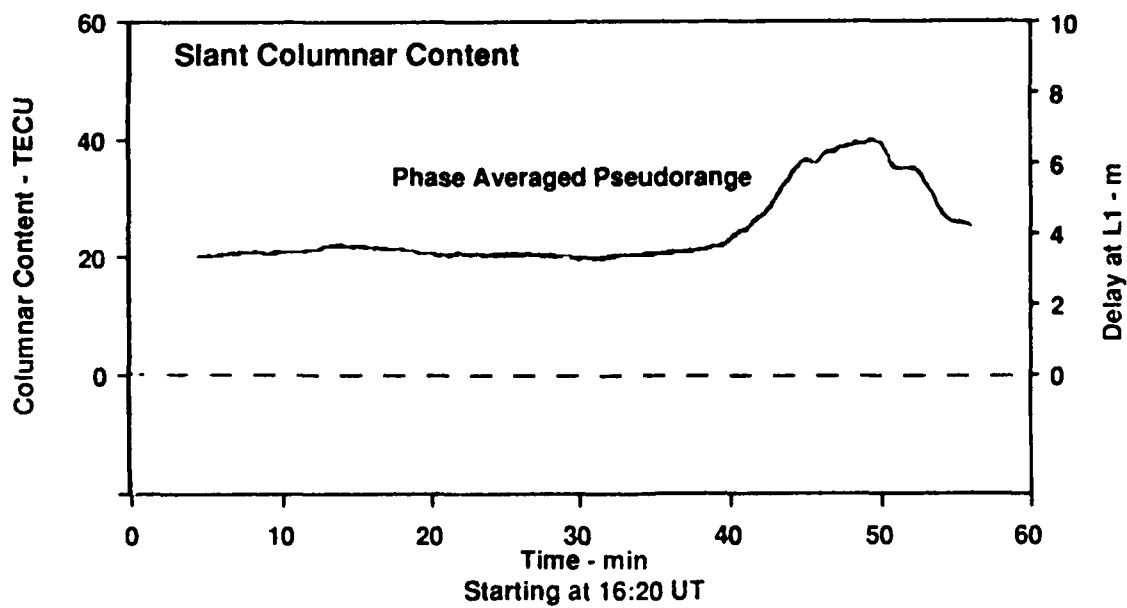
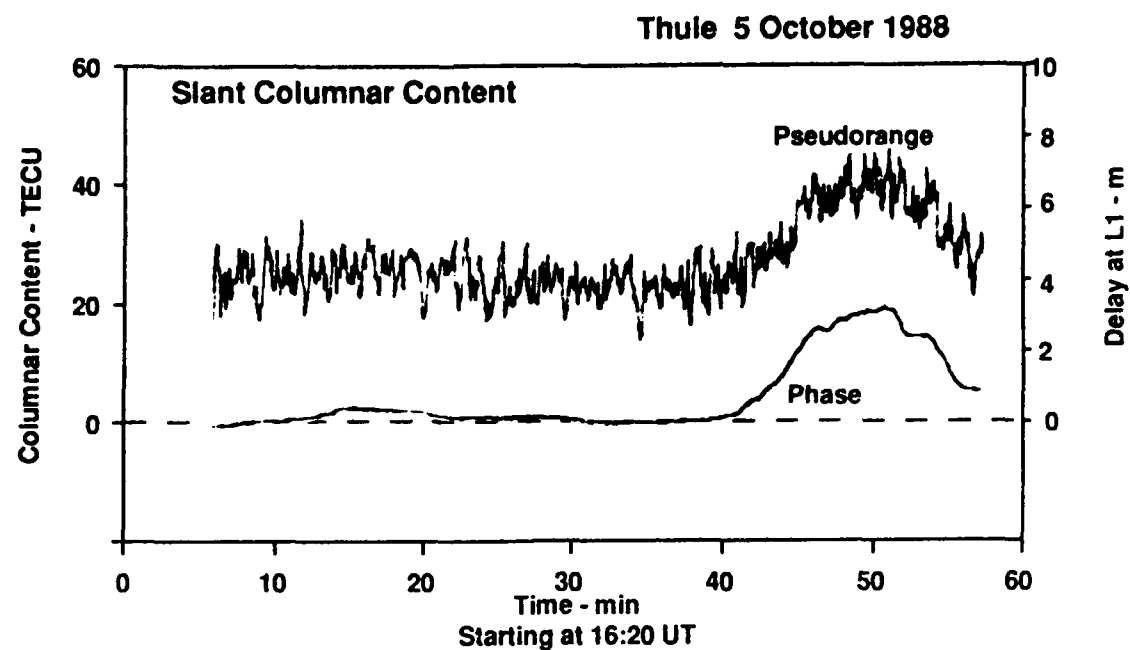
The pseudorange measurements are much noisier than the phase measurements, usually by a factor of 100 or more. Therefore, it is advantageous to use the phase data along with the pseudorange data. The phase data can be combined with the pseudorange to provide an estimate of the bias. This is equivalent to the phase averaged pseudorange techniques of geodetic positioning. If  $\Delta N_{cs}$  is the phase measurement (including the bias) and  $N_{cs}$  is the pseudorange measurement of the slant columnar content, then the bias is

$$B_{\phi} = (\Delta N_{cs} - N_{cs}) \quad ,$$

where the average is taken over all data in a period of continuous phase lock.

An example of this bias estimation process is shown in Fig. A.1. The top plot shows the pseudorange and phase measurements. There is no loss of phase lock during this period; therefore, one bias estimate is generated from





**FIGURE A.1  
COMBINING PSEUDORANGE AND PHASE MEASUREMENTS**

these data. The resulting phase averaged pseudorange slant TEC is shown in the bottom plot. This is simply the phase data with the bias added.

**APPENDIX B**  
**USING SLANT TEC MEASUREMENTS TO DEVELOP THE**  
**IONOSPHERIC MODEL**

In order to use slant TEC measurements through a time varying three-dimensional ionosphere to develop an ionospheric model some assumptions about the structure of the ionosphere are required. The time dependence is assumed to be removed by using an earth centered reference frame that co-rotates with the sun as discussed earlier in this report. The reduction of the slant TEC measurements to a two-dimensional model in this co-rotating reference frame is accomplished with two steps: the use of an obliquity factor to convert the slant TEC measurement to an equivalent vertical TEC measurement and the assignment of this vertical TEC value to a point in the two-dimensional reference frame. The implications of these two steps on the accuracy of the model are discussed in this Appendix.

This problem of reduction of slant measurements has been addressed in a somewhat different form by those workers who developed the M factor for deriving vertical TEC values from Faraday rotation measurements (Titheridge 1972, Davies et al. 1976). Titheridge found through simulations using typical ionospheric profiles that slant Faraday rotation measurements could be reduced to vertical TEC (integrated up to 2000 km) with an accuracy of 5% for most conditions. This study addressed the effects of the variation of the magnetic field as a function of height and the variation in the vertical electron density profiles on the M factor. The magnetic field effects do not play a role in the differential group delay measurements but the variation of the electron density profiles are an important factor.

The region of the ionosphere that is considered in this study for modeling the data from the Austin station is only a small fraction of the total global ionosphere. To get an idea of the relationship of this region to the global ionosphere we show in Fig. B.1 an ionospheric map of the vertical TEC derived from the Bent model (Bent et al. 1972) with the modeled region shown in the shaded area. This global model, of course, does not attempt to illustrate the small scale gradients but does give an idea of the average large scale gradients produced by solar radiation effects.

The most important input parameter for this model is the sunspot number. We have used a sunspot number of 20, which is the approximate sunspot number for the CETKE data time frame. We should note that this time period is

APRIL 0000 UT SUNSPOT NUMBER 20

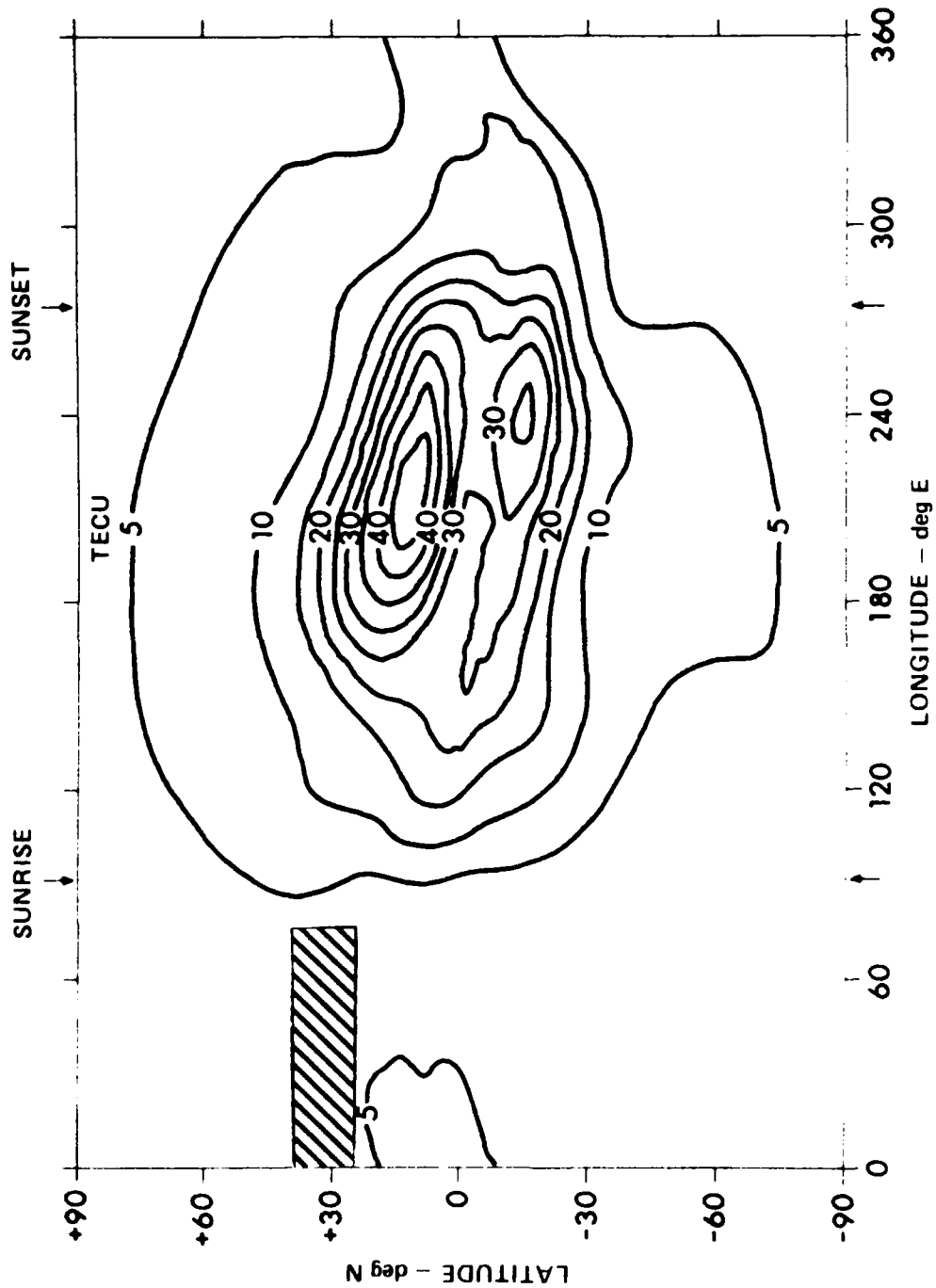


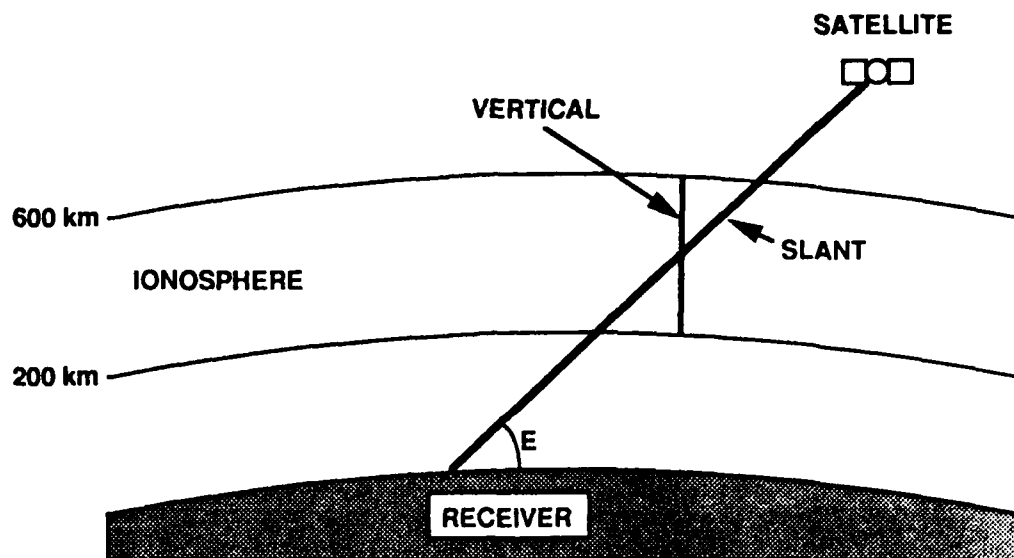
FIGURE B.1  
BENT IONOSPHERIC MODEL  
OF ELECTRON COLUMNAR CONTENT

near the minimum of the 11 year sunspot cycle and that sunspot numbers as large as 150 can occur during sunspot maximum. Global TEC models during sunspot maximum will show the same general features as those for sunspot minimum but the TEC levels will increase. The maximum TEC for a Bent model at the sunspot maximum will be greater than 100 TECU as compared to the maximum of around 50 TECU shown in this plot.

The Bent model suggests that the modeled region may be relatively smooth and probably will not involve any large horizontal gradients. If this holds for the actual ionosphere then we shall be able to convert slant TEC measurements to two-dimensional model points by making some assumptions about the ionospheric structure without incurring large errors.

The first step converts the slant TEC measurement to an equivalent vertical TEC value. To accomplish this it is necessary to make assumptions about the vertical electron density profile and the variation of this profile along the line of sight. In developing the obliquity factor model we have assumed an ionospheric profile in which all of the electrons are uniformly distributed between 200 and 600 km with no variation in this profile along the line of sight. This simple model is illustrated in Fig. B.2 and produces the obliquity factor shown in Fig. 2.1.

To get an idea of the errors incurred by using this obliquity factor model we compared this obliquity factor against a more realistic obliquity factor derived from Bent model ionospheric profiles. These profiles vary somewhat with geographic position and time but they all have the basic Chapman layer profile shape (Bent et al. 1972, Risbeth and Garriott 1969). The obliquity factor from the Bent profiles was obtained by integrating the electron density along the line of sight using a 1000 km circular polar orbit for this simulation. The results of this simulation should not vary significantly from the GPS orbit case because most of the electrons (and, more importantly, the horizontal gradients) are located below 1000 km. The Bent model in this simulation used a sunspot number of 80.



**FIGURE B.2**  
**OBLIQUITY FACTOR MODEL GEOMETRY**

This simulation was performed for 36 stations uniformly distributed throughout the world, using four azimuth directions and six elevation angles at each station. The maximum and root mean square (rms) deviation of the ratio between the complex Bent model and simple slab model obliquity factors for each station were calculated. If we consider the elevation angles  $30^\circ$  and above the maximum deviation of the ratio from unity over all stations, elevation and azimuth angles are less than  $6^\circ$  with rms of the deviation less than 3%. The errors increase with decreasing elevation angle with maximum deviation of 20% and rms of 10% at  $10^\circ$  elevation.

The results of this simulation show that the obliquity factor model can be used with a wide variety of realistic ionospheric profiles without introducing a large error. Similar obliquity factor models have been used by other workers (Lanyi and Roth 1988). It is usually not worthwhile to introduce a more complicated obliquity factor model unless additional information about the actual ionospheric profiles is available.

The second step of assigning the vertical TEC value to a point in the two-dimensional model is also related to the electron density profile. We have chosen the intersection of the line of sight with the ionosphere, with the ionosphere at 350 km as the point where the vertical TEC is to be assigned. The rationale behind the choice of 350 km is that this is a typical value for the height of the maximum electron density. If the electron density profiles were symmetric about the maximum and all the profiles were identical along the line of sight, then the mapping of the measurements to the two-dimensional model would be exact. In reality, the profiles are usually close enough to these ideals so that the two-dimensional model is a good estimate.

To get an idea of how the two-dimensional model could be distorted by these approximations, we performed a second simulation to investigate how the assigned position of the vertical TEC using a fixed 350 km intersection height differs from the "true" position. For this simulation, we used a typical mid-latitude Chapman model profile with a gradient in the TEC along the line of sight. The relative profile shapes were the same along the line of sight. The "true" vertical TEC position was calculated as the position with the same vertical



TEC value as that calculated from the slant TEC corrected by the obliquity factor model.

When the "true" positions are compared with the assigned positions the differences are less than 100 km for a range of gradients from 5-25% per degree of earth's central angle. If the gradient is very small (less than 5%) then the position differences will be larger but the corresponding TEC differences will be smaller. This implies that for realistic ionospheric conditions the position assignment error will be less than 100 km. If the ionosphere is very smooth the errors can increase above this level but then in this case the TEC errors will be very small.

The total extent of latitude modeled in this study with the CETKE data is on the order of  $15^\circ$  of latitude, which corresponds to about 1700 km. Thus, the maximum position errors would be a small fraction of the total extent.

These simulations are meant to demonstrate that the two steps required to use slant TEC measurements to develop a two-dimensional model are reasonable assumptions and do not introduce large errors. These errors will cause the two-dimensional model to be distorted somewhat from the true TEC contours. If the purpose of the two-dimensional model is solely to separate the SV and receiver biases from the ionospheric TEC, then the distorted model indirectly contributes to errors in the estimated biases. If, however, the two-dimensional model is used as an actual map of the ionospheric TEC, then these errors are directly related to the accuracy of the map.

## REFERENCES

- Brent, Rodney B., Sigrid K. Llewellyn, and Margaret K. Walloch, 1972. "Description and Evaluation of the Bent Ionospheric Model," Vol. 1, SAMSO Report No. TR-72-239, Space and Missile Systems Organization, Los Angeles, California.
- Coco, D. S., and J. R. Clynch, 1986. "System Performance Tests for the TI 4100 Geodetic Receiver," Proceedings of the Fourth International Geodetic Symposium on Satellite Positioning, Applied Research Laboratories, The University of Texas at Austin, Austin, Texas, 28 April - 2 May 1986.
- Davis, K., R. B. Fritz, and T. B. Gray, 1976. "Measurement of the Columnar Electron Contents of the Ionosphere and Plasmasphere," J. Geophys. Res. 81, No. 1, 2825.
- Lanyi, G. E., 1986. "Total Ionospheric Electron Content Calibration Using SERIES Global Positioning Satellite Data," TDA Progress Report 42-85, NASA-JPL, pp. 1-12.
- Lanyi, Gabor E., and Titus Roth, 1988. "A Comparison of Mapped and Measured Total Ionospheric Electron Content Using Global Positioning System and Beacon Satellite Operations," Radio Science 23, No. 4, 483-492.
- Lanyi, G. E., 1988. Private Communication.
- Rishbeth, Henry, and Owen K. Garriott, 1969. Introduction to Ionospheric Physics (Academic Press, New York).
- Texas Instruments, 1982. "TI 4100 NAVSTAR Owner's Manual," Lewisville, Texas, pp. 1-4.
- Titheridge, J. E., 1972. "Determination of Ionospheric Electron Content from the Faraday Rotation of Geostationary Satellite Signals," Planet. Space Science 20, 353-369.
- Ward, P. W., 1982. "An Advanced NAVSTAR Geodetic Receiver," Third International Geodetic Symposium on Satellite Doppler Positioning, Las Cruces, New Mexico, p. 1123.
- Winn, F. B., 1988. (The Aerospace Corporation) Private Communication.